COMPUTATION OF LIFE EXPECTANCY OF AERONAUTICAL STRUCTURES

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abstract : *Ageing aircraft structures are susceptible to a very typical and dangerous form of fatigue damage named "Multiple Site Damage". An overall assessment method has been developed in order to improve the understanding of this phenomenon and to provide inspection programs. A full deterministic approach has been completed first. An automatic numerical process, using the Dual Boundary Element Method and Fracture Mechanics laws, computes the crack evolution until failure of the structure. Then, in order to take materials scatter into account, a probabilistic approach has been developed based on Monte-Carlo simulations. Initial cracks scenarios are randomly defined and cracks evolution is computed for each of them. Thus statistical results are provided. The results obtained are in good agreement with experimental results carried out at the AEROSPATIALE MATRA research center.*

1. INTRODUCTION

Multiple Site Damage (MSD) is a type of cracking that may be found in ageing airplanes which can adversely affect the damage tolerance of an airframe structure. It is defined as the simultaneous occurence of fatigue cracks in the same structural element. This phenomenon generally occurs in areas where a lot of critical sites (fastener holes for example) are subjected to the same stress level, as in the riveted joints of a fuselage. Multi-site crack growth can greatly weaken any part of the structure.

The occurrence of MSD in older aircraft was highlighted by the in-flight failure of a portion of the fuselage of an Aloha Airlines Boeing 737 in April 1988 (Fig. 1). At the time of the incident, the plane had accumulated nearly 90000 flights. The failure was precipitated by the linkup of small fatigue cracks emanating from adjacent rivet holes in the lap joint of the fuselage [1].

Currently, approximately 50% of the jet airplanes in the fleet are over 15 years old. The increasing number of in-service ageing aircraft as well as new forthcoming airworthiness requirements have incited manufacturers to develop methods to better understand the behavior of multiple cracked structures. The main objective is to provide engineering models to assess MSD regarding the problems of maintenance programs of ageing aircraft as well as the design of new ones.

Fig. 1. In-flight failure of a portion of the fuselage of an Aloha Airlines Boeing 737 [1].

As in the case of single crack, the MSD problem may be divided into different phenomena: Crack initiation, crack propagation and residual strength. The main particularity in the case of MSD is that these three stages are strongly linked. Behavior of one crack will depend on the behavior of adjacent cracks. Residual strength of the structure will be deeply influenced by the initial cracks configuration and the development of cracks patterns.

In this paper, an overall assessment method has been developed in order to improve the understanding of this phenomenon. A full deterministic approach has been completed first. An automatic numerical process, using the Dual Boundary Element Method and Fracture Mechanics laws, computes the crack evolution until failure of the structure. Then, in order to take materials scatter into account, a probabilistic approach has been developed based on Monte-Carlo simulations. Initial cracks scenarios are randomly defined and cracks evolution is computed for each of them. Thus statistical results are provided.

2. DUAL BOUNDARY INTEGRAL EQUATIONS

2.1 Formulation

The dual equations, on which the DBEM is based, are the displacement and the traction boundary integral equations (2,3). In the absence of body forces, the boundary integral representation of the displacement components u_i, at a boundary point P , not on crack surfaces, is given by

$$C_{ij}(P)\, u_j(P) + \int_{\partial\Omega} T_{ij}(P,Q)\, u_j(Q)\, ds(Q) = \int_{\partial\Omega} U_{ij}(P,Q)\, t_j(Q)\, ds(Q) \tag{1}$$

where $\partial\Omega$ indicates the problem boundary (including the crack faces), i and j denote Cartesian components, $T_{ij}(P,Q)$ and $U_{ij}(P,Q)$ represent the Kelvin traction and displacement fundamental solutions, respectively, at a boundary point Q.

The stress components σ_{ij} are obtained by differentiation of equation (1), followed by the application of Hooke's law; they are given by

$$C_{ij}(P)\, \sigma_{ij}(P) + \int_{\partial\Omega} S_{ijk}(P,Q)\, u_k(Q)\, ds(Q) = \int_{\partial\Omega} D_{ijk}(P,Q)\, t_k(Q)\, ds(Q) \tag{2}$$

and the traction components, t_j , are given by

$$C_{ij}(P)\,t_j(P) + n_i(P)\int_{\partial\Omega} S_{ijk}(P,Q)\,u_k(Q)\,ds(Q) = n_i(P)\int_{\partial\Omega} D_{ijk}(P,Q)\,t_k(Q)\,ds(Q) \tag{3}$$

where n_i denotes the ith component of the unit outward normal to the boundary, at point P. If P is on the crack surfaces, the displacement equation becomes :

$$C_{ij}(P^+)u_i(P^+) + (1 - C_{ij}(P^+)u_i(P^-\ )\int_{\partial\Omega} T_{ij}(P^+,Q)u_j(Q)\,ds(Q) = \int_{\partial\Omega} U_{ij}(P^+,Q)t_j(Q)\,ds(Q) \tag{4}$$

and the traction equation becomes :

$$C_{ij}(P^+)t_i(P^+) + C_{ij}(P^-)t_i(P^-\ ) + n_i(P^+)\int_{\partial\Omega} S_{ijk}(P^+,Q)\,u_k(Q)\,ds(Q) = n_i(P^+)\ \int_{\partial\Omega} D_{ijk}(P^+,Q)\,t_k(Q)\,ds \tag{5}$$

The crack-tip is modeled by singular elements that exactly represent the strain field singularity $1/\sqrt{r}$. The nodes are positioned at $\xi = -2/3$, $\xi = 0$ and $\xi = 2/3$.

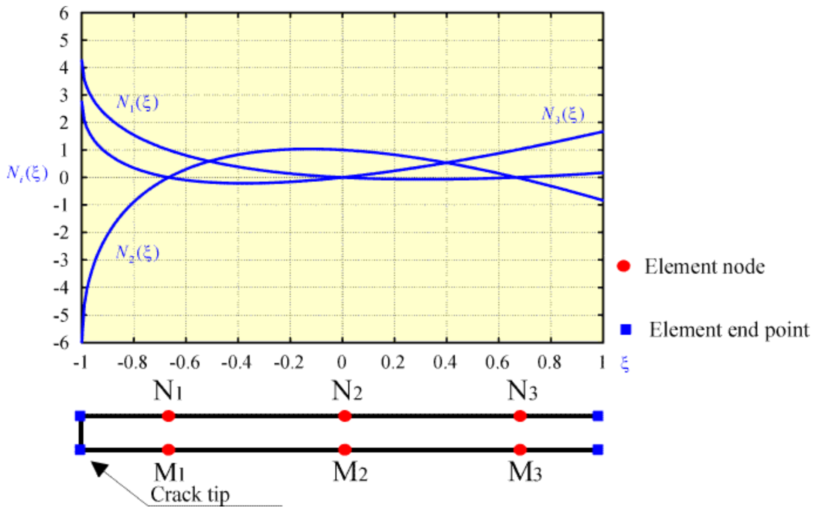


Fig. 2. Crack-tip elements.

2.2 The stress intensity factor evaluation

There are several approaches to compute S.I.F. using a dual boundary element formulation. In the present work, they are computed from the crack opening displacements at nodes which are extremely close to the tip.

$$K_I = \frac{\mu}{k+1}\sqrt{\frac{\pi}{l}} \quad [5(u_2^{N_2} - u_2^{M_2}) - \frac{3\sqrt{5}}{5}(u_2^{N_3} - u_2^{M_3})] \tag{6}$$

and

$$K_{II} = \frac{\mu}{k+1}\sqrt{\frac{\pi}{l}} \quad [5(u_1^{N_2} - u_1^{M_2}) - \frac{3\sqrt{5}}{5}(u_1^{N_3} - u_1^{M_3})] \tag{7}$$

where μ is the shear modulus and $k = 3 - 4\eta$; for plane strain $\eta = \nu$ and for plane stress $\eta = \frac{\nu}{(\nu+1)}$, where ν is the Poisson's ratio.

2.3 Crack modeling strategy

The general modeling strategy can be summarized as follows [4] :

- all boundaries are modeled with discontinuous quadratic elements, except the crack-tip which is modeled by a singular element;
- the displacement equation (4) is applied to collocation nodes on one of the crack surfaces;
- the traction equation (5) is applied to collocation nodes on the other crack surface;
- the displacement equation (1) is applied to collocation nodes on all non-crack boundaries
- the Stress Intensity Factors are computed from the Crack OpeningDisplacement at collocation points of the singular element.
- the new crack-extension increment is modeled with new singular boundary elements. They will generate new equations and update the ones already existing with new unknowns.

3. SIMULATION MODEL

There are different approaches to take into account the probabilistic character of the MSD problem. However, in the present paper, a mixed model is used, i.e. a probabilistic approach is used for the determination of a starting fatigue life scenario; while the subsequent steps (e.g. the damage accumulation, crack growth and crack coalescence) are calculated by a deterministic model.

3.1 Crack initiation

In this model, it is assumed that the mean value of the fatigue life and the scatter factor (the standard deviation) for a single point are known (Wöhler curve Fig. 3)

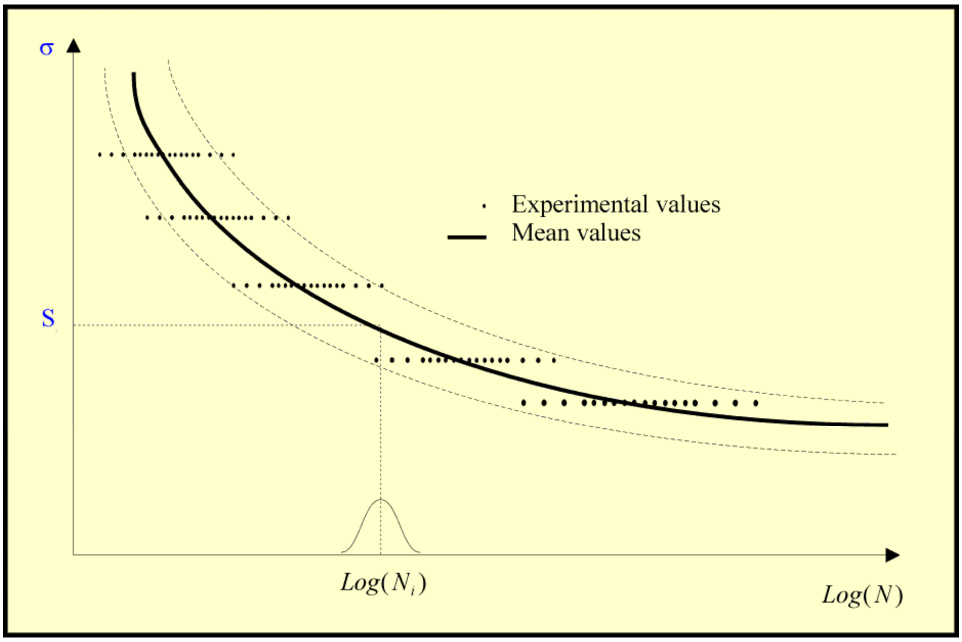


Fig. 3 Wöhler curve (S-N).

For the aluminum 2024 T3 alloy the mean value of the stress load initiation cycles is :

$$N = 10^5 \left(\frac{S - S_{\lim}}{IQF - S_{\lim}}\right)^P \tag{8}$$

where :

$$p = 2.28 \qquad IQF = 176MPa \qquad S_{\lim} = 59MPa \tag{9}$$

and the standard deviation is:

$$\sigma = \frac{a}{S^2} + \frac{b}{S} + c \tag{10}$$

where:

$$a = 2169.9 \qquad b = 1.2987 \qquad c = 0.0279 \tag{11}$$

The fatigue life is distributed according to a log-normal distribution, this may be done by means of an ordinary random number in the interval [0; 1], and by inversion of the log-normal distribution by an approximate method.

Once the fatigue life N_i of all fatigue critical locations "i" (i = 1; M) has been determined, we compute the damage accumulation "D_i" in each location "i" as :

$$D_i = \frac{N_k}{N_i} \tag{12}$$

where : $N_k = Min\,(N_i) \quad (i = 1, M)$

It is assumed that the fatigue damage at the location "k" is high enough to start a crack growth while all other locations "i" ($i = 1, M$ and $i \neq k$) have to accumulate more fatigue damage until the crack starts (D_i = 1).

The origins of the new cracks are located at boundaries where the damage accumulation D_i is equal to 1, the new cracks are given a small finite initial length a_0 and a direction that is perpendicular to the boundary. They are then allowed to grow during subsequent loading steps.

3.2 Propagation

Generally, the crack propagation is calculated by means of the Stress Intensity Factors (K_I and K_{II}) and the Paris equation :

$$\frac{da}{dN} = C(\Delta K)^n) \tag{13}$$

where da is the increment of crack growth in dN stress cycles. The parameters C and n are material constants, and ΔK is the range ($K_{\max} - K_{\min}$) of the stress intensity factor during these cycles.

3.3 coalescence

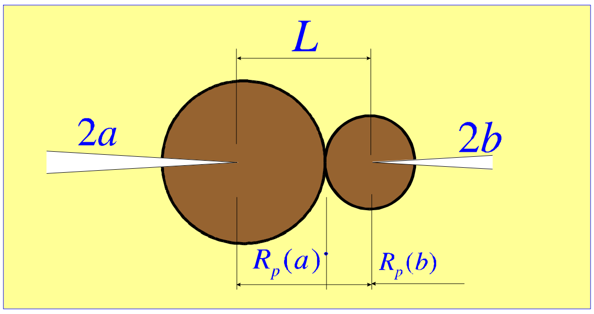


Fig. 4. Touching plastic zones as a criterion for ligament failure.

In this model mainly the detection of the link-up of relatively small cracks is essential. This may be treated in different ways. One of the very simple models for this problem seems to be the model proposed by Swift. It actually checks a criterion which is very similar to a "net section yielding" criterion.

The main idea is to use the radius of the plastic zone in front of each crack tip

$$R_p = \frac{1}{2\Pi}[\frac{K_{eff}}{\sigma_{0.2}}] \tag{14}$$

and to apply the contact of both plastic zones as a criterion for the link-up of the cracks:

$$R_p(a) + R_p(b) = L \tag{14}$$

4. Algorithm

The overall problem of the assessment of MSD in aircraft structures are approached by Monte-Carlo simulation. Initial crack scenarios are defined using a random process. For each of them, a deterministic computation of crack growth until fracture is completed. Detection

probability aspect is also taken into account in the whole process. Figure 6 shows a diagram of the general approach implementation.
We present here the overall structure of the model, where "a" indicates the crack length and D indicates the accumulated damage.

- Determination of the M fatigue critical locations and the values of the stresses $S_i\,(i = 1;\ M)$
- Loop over "n" scenarios
  - random run for the determination of the initiation number of load cycles N_i for each site i by using the Wöhler curve and the value of S_i.
  - Compute the initial damage : $D_i = \dfrac{N_k}{N_i}$

    where $N_k = Min\,(N_i)$ $(i = 1, M)$
  - Introduce a small crack a_0 in the site k
  - loop over the load cycles "l"
    - loop over all sites "i"

      if the location is still in the damage accumulation phase ($D_i < 1$) then :

      - compute the new stress $S_i^{'}$
      - $D_i = D_i + \dfrac{\Delta N}{N_i'}$

      (where $N_i^{'}$ is computed by using the Wöhler curve and the value of $S_i^{'}$)

      if $D_i = 1$ then: introduce a new small crack a_0 in the site "i"

      endif

      else if the location is an active crack then:

      - calculation of the Stress Intensity Factor K_{eff}
      - $\Delta a_i = C(\Delta\ K_{eff})^n\ \Delta N$
      - $a_i = a_i + \Delta a_i$
      - calculation of plastic zone size
      - check link-up criterion

      endif
    - check overall residual strength of the structure
  - calculation of the life expectancy for a given configuration
- calculate statical values for the problem: mean value, standard deviation, etc.

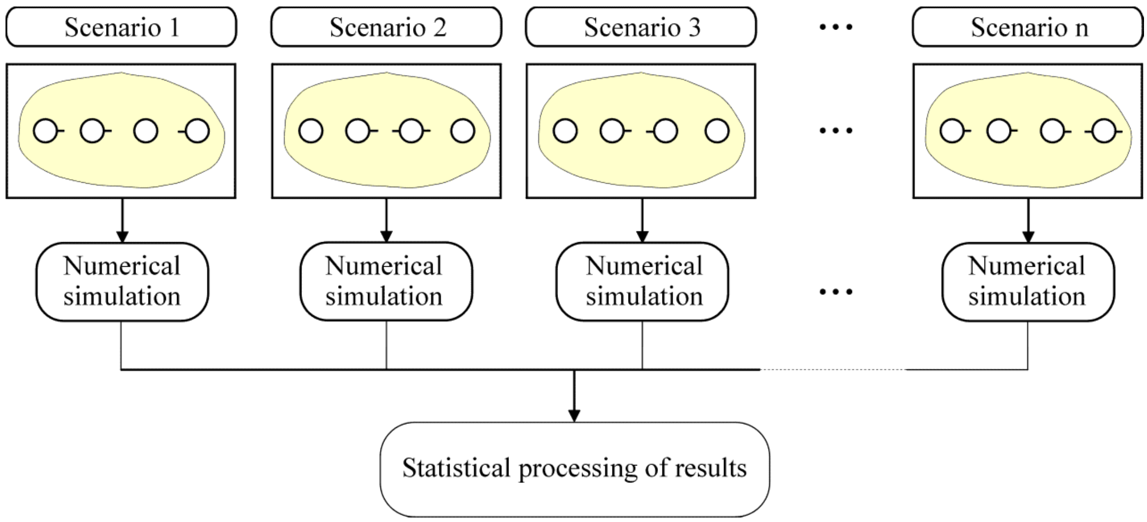


Fig. 5. Gneral approach.

To illustrate our approach, we consider the example of a rectangular plate with two holes. We present the two first scenarios of the Monte-Carlo simulation in the following.

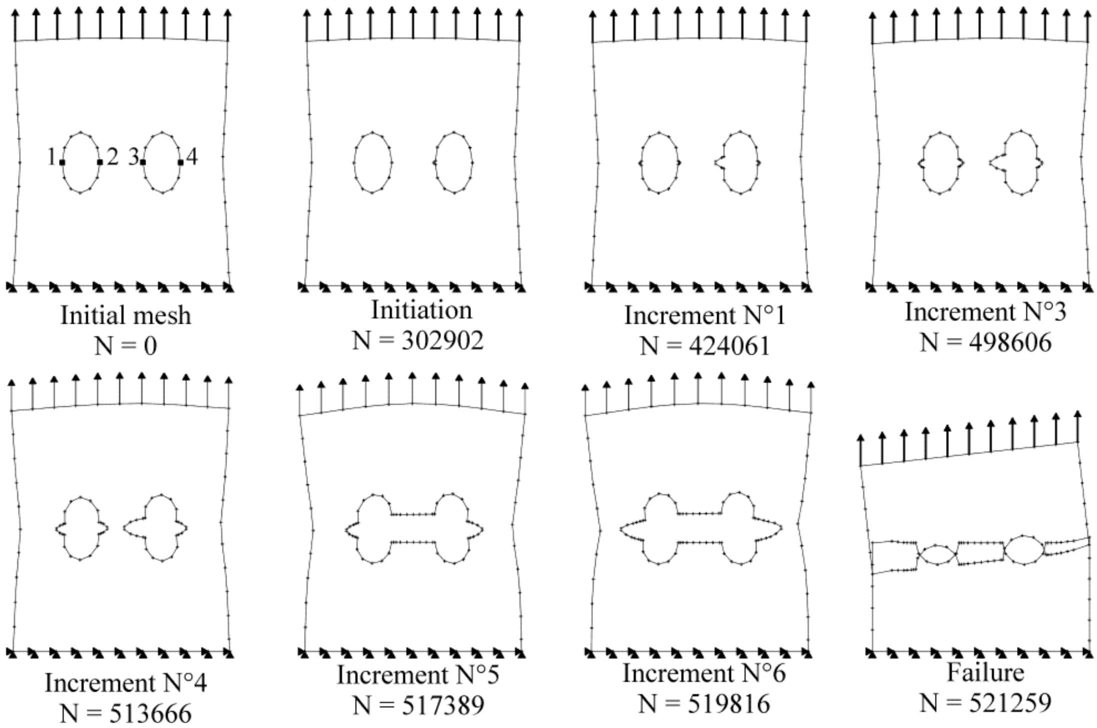


Fig. 7. First scenario of Monte-Carlo Simulations

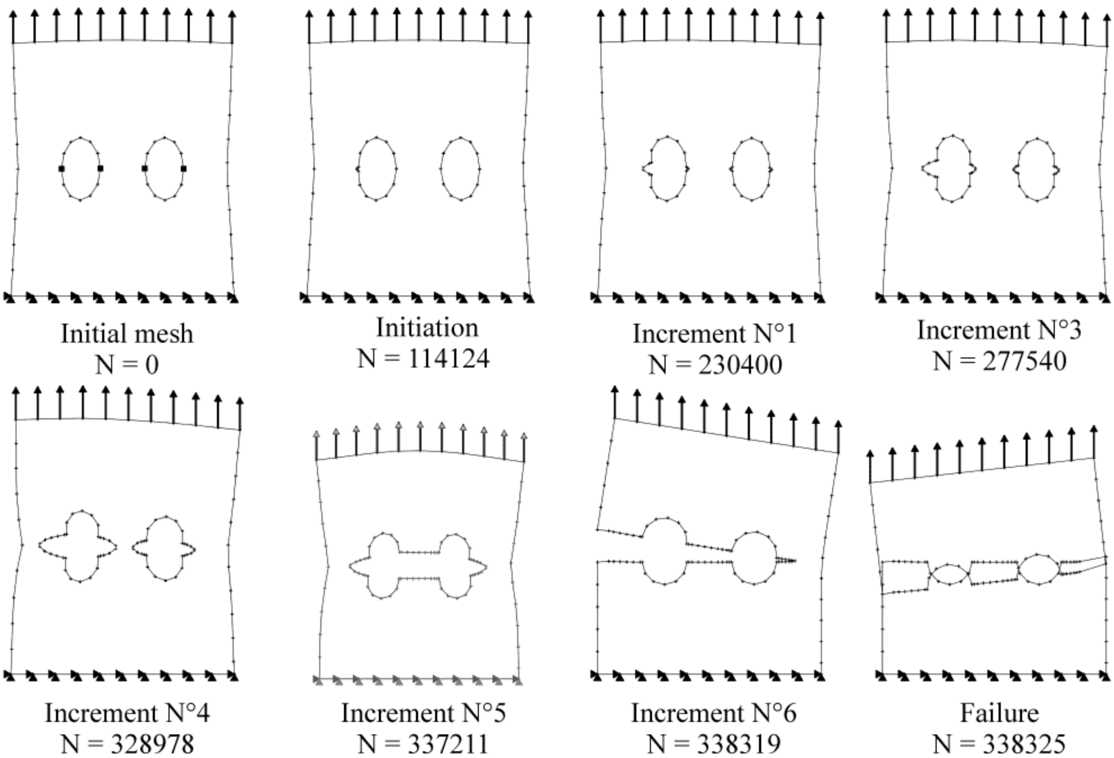


Fig. 8 Second scenario of Monte-Carlo Simulations

5 NUMERICAL RESULTS

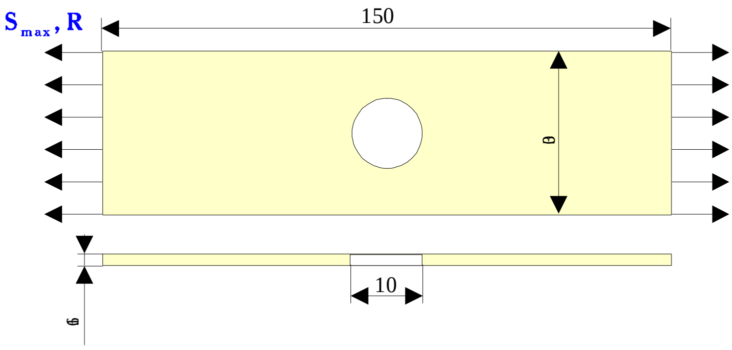
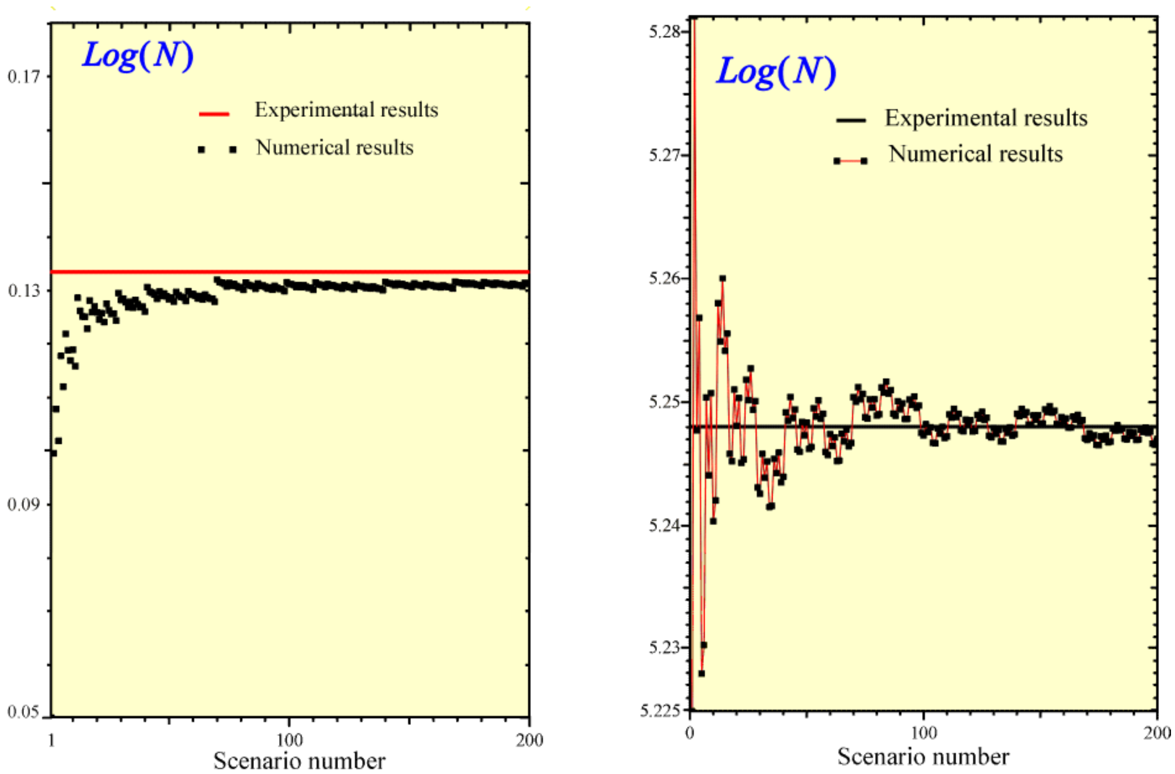


Fig. 9. Rectangular plate with one hole.

Consider a rectangular plate with one hole as shown in figure (Fig. 9). Initiation and crack growth tests were carried out at AEROSPATIALE MATRA research center under R = 0.1 constant amplitude loading at a frequency of 15 Hz ($S_{\max} = 100MPa$).

The numerical results were collected for 200 different configurations, the mean value and the standard deviation of life expectancy were compared with the experimental one. We note that the results given by this model are in good agreement with the experimental values (Fig. 10).



Mean value of life expectancy

Standard deviation of life expectancy

Fig. 10. Statistical results of life expectancy.

6 CONCLUSION

An overall assessment method has been developed in order to improve the understanding of MSD phenomenon. A full deterministic approach has been completed first. An automatic numerical process, using the Dual Boundary Element Method and Fracture Mechanics laws, computes the crack evolution until failure of the structure. Then, in order to take materials scatter into account, a probabilistic approach has been developed based on Monte- Carlo simulations. The results obtained are in good agreement with experimental results carried out at the AEROSPATIALE MATRA research center. The work on this method will be continued in order to refine and extend the model to the bolted joints.

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